Phase transitions, hysteresis and bifurcation for generalized Mooney-Rivlin elastic bodies

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We present a simple model capable of qualitatively describing phase transitions, hysteresis and bifurcation issues for isotropic, incompressible elastic bodies subject on the boundary to a homogeneous distribution of dead-load surface tractions corresponding to a simple shear stress accompanied by a uniaxial traction. According to the constitutive assumption developed in [1], we study solid phase transformations by considering a generalized Mooney-Rivlin material whose strain energy density is not rank-one convex on the class of simple shear deformations. As pointed out in [2], the study of the local stability of single-phase homogeneous equilibrium deformations is strongly related to the analysis both of the *strong ellipticity* condition for the incremental elasticity tensor at internal points and of the *complementing condition* for the incremental elasticity tensor at points on the free boundary. Here, the study of the complementing condition is based on the solutions of a Riccati algebraic matrix equation whose known matrices are defined by the incremental elasticity tensor at points on the free boundary (cf. [3], [4]).

We further analyze the response of the body in a quasi-static loading-unloading process and show that there exists exactly one “critical” value of the prescribed load which *may support* stable two-phase deformations, that is pairs of stable equilibrium deformations which satisfy the Hadamard rank-one compatibility condition across the discontinuity surface between the phases. According to a classical entropy criterion (cf. [5]), we then check the actual emergence of coexisting phases by evaluating the sign of the *driving traction* acting on the discontinuity surface, that is the sign of the difference of the total energy density of the two phases. It is worth noting that during the whole loading-unloading process we obtain a *hysteresis loop* in the load-strain diagram which shows good agreement with experimental data from [6].

Finally, following the approach developed in [7], we show the existence of a *local branch of diffuse bifurcating modes* during the loading process.

References

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