A one-dimensional variational model for fracture and plasticity, based on diffuse cohesive energy

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Abstract

The variational model proposed in [Del Piero, Lancioni, March, A diffuse cohesive energy approach to fracture and plasticity: the one-dimensional case, pre-print, 2012] for the quasi-static evolution of the inelastic deformation of a bar subjected to uniaxial stretching is presented and discussed. The energy functional is assumed to be the sum of three terms: an elastic bulk energy, an inelastic cohesive energy, and a non-local gradient term.

The model describes fracture as the final stage of plastic deformation. A similar model describing fracture produced by damage has been proposed by [Pham, Amor, Marigo and Maurini, 2011]. For both models, a major difference from the variational model of [Francfort and Marigo 1998] is that the inelastic energy, instead of being concentrated on singular surfaces, is assumed to be diffused over the volume. This corresponds to assuming that fracture is preceded by the formation of a process zone, in which the material becomes weaker and more deformable. The main difference between the plasticity and the damage model is that in the first the energetic contributions of the elastic and inelastic deformation are separated, while in the second the inelastic deformation affects the elastic properties of the material.

The evolution of the bar's response under increasing elongation produced by prescribed boundary displacements is determined by incremental energy minimization, constrained by a dissipation inequality. The phenomenological assumptions of classical plasticity, such as yield condition, hardening rule, consistency condition, and elastic unloading, here come as necessary conditions for a minimum.

Depending on the analytical form assumed for the cohesive energy, a large variety of responses is predicted. Indeed, a convex cohesive energy produces a work-hardening response, with inelastic deformations almost homogeneously distributed over the bar. A concave cohesive energy may produce both hardening and softening response, with or without strain localization, as well as the overcoming of catastrophic fracture, depending on the interplay of some material parameters, interpreted as internal lengths of the material, and the length of the bar. In the case of a concave cohesive energy, the convexity/concavity of its derivative determines two basic types of rupture mechanisms, typical of steel and concrete, respectively. In the first case the inelastic deformation concentrates on a singular surface and produces a catastrophic fracture, while in the second case the inelastic deformation eventually spreads over the bar, producing a gradual loss of strength without any catastrophic event.

In the proposed talk, the main results of the incremental energy minimization is first described. Then, a closed-form solution is presented for the incremental response at the onset of the inelastic deformation. Finally, the theoretical results are compared with experiments and with results of numerical simulations.